DAY TWENTY ONE

# Magnetic Effect of Current

## Learning & Revision for the Day

- Concept of Magnetic Field
- Biot-Savart's Law and its Applications
- Ampere's Circuital Law
- Force on a Moving Charge in Uniform Magnetic Field
- Cyclotron
- Magnetic Force on a Current Carrying Conductor
- Moving Coil Galvanometer

## Concept of Magnetic Field

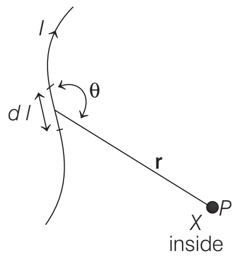
If a magnet is placed in a magnetic field, then it experiences a force on it, Also, when a magnet is placed near a current carrying conductor, then it experiences the similar force, it means that current carrying conductor produces a magnetic field around it. This effect of current is called **magnetic effect of current**.

## Biot-Savart's Law and its Applications

The magnetic field $d\mathbf{B}$ at a point $P$, due to a current element $I\,d\mathbf{l}$ is given by

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I(d\mathbf{l} \times \mathbf{r})}{r^3}$$

where, $\theta$ is the angle between $d\mathbf{l}$ and $\mathbf{r}$.



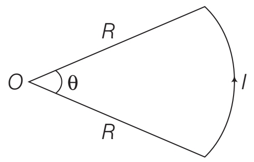

Direction of magnetic field produced due to a current carrying straight wire can be obtained by the right hand thumb rule.

## Magnetic Field due to Circular Current Loop

- If there is a circular coil of radius $R$ and $N$ number of turns, carrying a current $I$ through the turns, then magnetic field at the centre of coil is given by

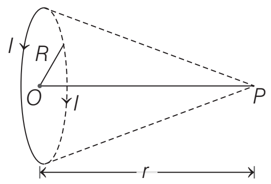
$$B = \frac{\mu_0 NI}{2R}$$

- If there is a circular arc of wire subtending an angle $\theta$ at the centre of arc, then the magnetic field at the centre point




$$B = \frac{\mu_0 I}{2R}\left(\frac{\theta}{2\pi}\right)$$

- At a point $P$ situated at a distance $r$ from centre of a current carrying circular coil along its axial line.




The magnetic field is

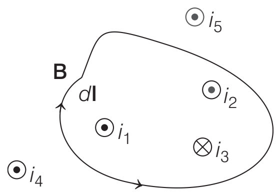
$$B = \frac{\mu_0 NIR^2}{2(R^2 + r^2)^{3/2}}$$

If $r >> R$, then at a point along the axial line, $B = \frac{\mu_0 NIR^2}{2r^3}$

## Ampere's Circuital Law

The line integral of the magnetic field $\mathbf{B}$ around any closed path is equal to $\mu_0$ times the net current $I$ threading through the area enclosed by the closed path.

Mathematically, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \Sigma I$




Now, consider the diagram above.

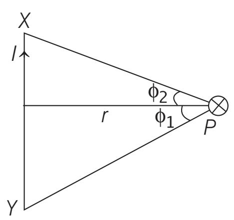
Here, $\Sigma I = i_1 + i_2 - i_3$

Hence, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \cdot (i_1 + i_2 - i_3)$

## Applications of Ampere's law

### 1. Magnetic field due to Straight Current Carrying Wire

The magnetic field due to a current carrying wire of finite length at a point $P$ situated at a normal distance $r$ is




$$B = \frac{\mu_0 I}{4\pi r}(\sin \phi_1 + \sin \phi_2)$$

- If point $P$ lies symmetrically on the perpendicular bisector of wire $XY$, then $\phi_1 = \phi_2 = \phi$ (say) and hence

$$B = \frac{\mu_0 I}{4\pi r} \cdot 2 \sin \phi = \frac{\mu_0 I \sin \phi}{2\pi r}$$

- For a wire of infinite length $\phi_1 = \phi_2 = 90°$ and hence

$$B = \frac{\mu_0 I}{2\pi r}$$

- When the wire $XY$ is of infinite length, but the point $P$ lies near the end $X$ or $Y$, then $\phi_1 = 0°$ and $\phi_2 = 90°$ and hence, $B = \frac{\mu_0 I}{4\pi r}$
- When point $P$ lies on axial position of current carrying conductor, then magnetic field at $P$,

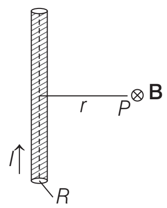
$$B = 0.$$

- When wire is of infinite length, then magnetic field near the end will be half, that of at the perpendicular bisector.

### 2. Magnetic Field due to a Thick (Cylindrical) Wire

Magnetic field at a point outside the wire

$B = \frac{\mu_0 I}{2\pi r}$, where $r$ is the distance of given point from centre of wire and $r > R$.




**Thick cylindrical wire**

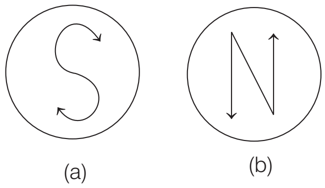
- Magnetic field at a point inside the wire at a distance $r$ from centre of wire ($r < R$) is

$$B = \frac{\mu_0 I}{2\pi} \cdot \frac{r}{R^2}$$

- Magnetic field inside a hollow current carrying conductor is zero.

### 3. Magnetic Field due to a Solenoid

A current carrying solenoid behaves as a bar magnet. The face, where current is flowing clockwise behaves as South pole and the face, where current is seen flowing anti-clockwise, behaves as North pole.




For such a solenoid, the magnetic field inside it is uniform and directed axially.

- For a solenoid coil of infinite length at a point on its axial line, the magnetic field, $B = \mu_0 nI$

  where, $n$ is number of turns per unit length.
- At the end of solenoid, $B = \frac{1}{2}\mu_0 nI$
- At the end field is half of at the centre, this is called **end effect**.

### 4. Toroidal Solenoids

For a toroid (i.e. a ring shaped closed solenoid) magnetic field at any point within the core of toroid $B = \mu_0 nI$,

where $n = \frac{N}{2\pi R}$, $R$ = radius of toroid.

## Force on a Moving Charge in Uniform Magnetic Field and Electric Field

- If a charge $q$ is moving with velocity $\mathbf{v}$ enters in a region in which electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ both are present, it experiences force due to both fields simultaneously. The force experienced by the charged particle is given by the expression

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) + q\mathbf{E}$$

Here, magnetic force $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = Bqv\sin\theta$ and electric force $\mathbf{F}_e = q\mathbf{E}$.

- The direction of magnetic force is same as $\mathbf{v} \times \mathbf{B}$ if charge is positive and opposite to $\mathbf{v} \times \mathbf{B}$, if charge $q$ is negative.

### Motion of a Charged Particle in a Uniform Magnetic Field

- (i) If a charge particle enters a uniform magnetic field $B$ with a velocity $v$ in a direction perpendicular to that of $B$ (i.e. $\theta = 90°$), then the charged particle experiences a force $F_m = qvB$. Under its influence, the particle describes a circular path, such that

**Radius of circular path**, $r = \dfrac{mv}{qB}$

In general,

$$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2mqV}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

where, $p = mv$ = momentum of charged particle, $K$ = kinetic energy of charged particle and $V$ = accelerating potential difference.

(ii) The **period of revolution** of charged particle $T = \dfrac{2\pi m}{qB}$,

the **frequency of revolution** $\nu = \dfrac{qB}{2\pi m}$

or **angular frequency** $\omega = \dfrac{qB}{m}$.

- (i) If a charged particle is moving at an angle $\theta$, to the magnetic field (where $\theta$, is other than 0°, 90° or 180°), it describes a helical path, where **radius of helical path**,

$$r = \frac{mv\sin\theta}{qB}.$$

(ii) **Revolution period**, $T = \dfrac{2\pi m}{qB}$

or **Frequency**, $\nu = \dfrac{qB}{2\pi m}$

(iii) Moreover, pitch (the linear distance travelled during one complete revolution) of helical path is given by

$$p = v\cos\theta \cdot T = \frac{2\pi mv\cos\theta}{qB}$$

- If the direction of a $\mathbf{v}$ is parallel or anti-parallel to $\mathbf{B}$, $\theta = 0$ or $\theta = 180°$ and therefore $F = 0$. Hence, the trajectory of the particle is a straight line.

If the velocity of the charged particle is not perpendicular to the field, we will break the velocity in parallel ($v_{||}$) and perpendicular ($v_{\perp}$) components.

$$r = \frac{mv_{\perp}}{qB}$$

$$\text{Pitch, } p = (v_{||})T$$

## Cyclotron

It is a device used to accelerate positively charged particles, e.g. proton, deuteron, $\alpha$-particle and other heavy ions to high energy of 100 MeV or more.

Cyclotron frequency, $\nu = \dfrac{Bq}{2\pi m}$.

Maximum energy gained by the charged particle

$$E_{max} = \left[\frac{q^2B^2}{2m}\right]r^2$$

where, $r$ = maximum radius of the circular path followed by the positive ion.

Maximum energy obtained by the particle is in the form of kinetic energy.

## Magnetic Force on a Current Carrying Conductor

If a current carrying conductor is placed in a magnetic field $\mathbf{B}$, then a small current element $I\,d\mathbf{l}$ experiences a force given by

$$d\mathbf{F}_m = Id\mathbf{l} \times \mathbf{B}$$

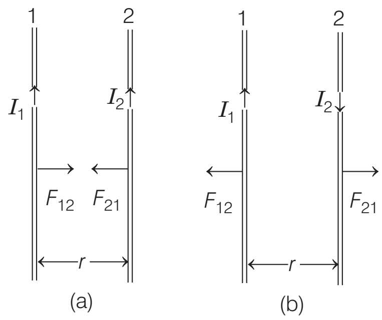
and the total force experienced by whole current carrying conductor will be

$$\mathbf{F}_m = \int d\mathbf{F}_m = \int I(d\mathbf{l} \times \mathbf{B})$$

The direction of force can also be determined by applying Fleming's left hand rule or right hand thumb rule.

### Force between Two Parallel Current Carrying Conductors

- Two parallel current carrying conductors exert magnetic force on one another.
- Magnetic force experienced by length $l$ of any one conductor due to the other current carrying conductor is




$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1I_2}{r} l$$

Force per unit length, $\dfrac{F}{l} = F_0 = \dfrac{\mu_0}{4\pi} \cdot \dfrac{2I_1I_2}{r}$

**NOTE**
- If the conductors carries current in same direction, then force between them will be attractive.
- If the conductor carries current in opposite direction, then force will be repulsive.

## Torque

When a current carrying loop placed in a uniform magnetic field, it experience torque,

$$\tau = NIAB \sin\theta$$

where, $NiA$ is defined as the magnitude of the dipole moment of the coil

$$(p_m) \cdot \tau = p_m B \sin\theta$$

$$\Rightarrow \qquad \tau = \mathbf{p}_m \times \mathbf{B}$$

**NOTE**
- A current carrying loop (of any shape) behaves as a magnetic dipole whose magnetic moment is given by
$$(p_m) = IA$$
- If we have a current carrying coil having $N$ turns, then magnetic moment $P_m$ of dipole will be
$$(p_m) = NIA$$
- Magnetic moment of a current carrying coil is a vector and its direction is given by right hand thumb rule.

# **Moving Coil Galvanometer** (MCG)

MCG is used to measure the current upto nanoampere. The deflecting torque of MCG,

$$\tau_{\text{def}} = NBIA$$

A restoring torque is set up in the suspension fibre. If $\alpha$ is the angle of trust, the restoring torque is

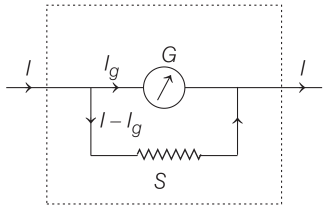
$$\tau_{\text{restoring}} = KI$$

where, $K$ is galvanometer constant.

## Some Important Concepts Related to Moving Coil Galvanometer

Some of the important concepts related to galvanometer, i.e. current sensitivity, voltage sensitivity and some of conversions used in galvanometer are given below.

- **Conversion of Galvanometer into Ammeter** An ammeter is made by connecting a low resistance $S$ in parallel with a pivoted type moving coil galvanometer $G$. $S$ is known as shunt.




Ammeter

Then, from circuit,

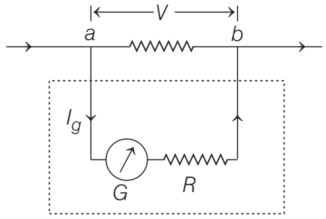
$$I_g \times G = (I - I_g) \times S$$

$$\Rightarrow \qquad S = \left(\frac{I_g}{I - I_g}\right) G$$

So, $S << G$, only a small fraction of current goes through the galvanometer.

- **Conversion of Galvanometer into Voltmeter**

A voltmeter is made by connecting a resistor of high resistance $R$ in series with a pivoted type moving coil galvanometer $G$.




Voltmeter

From the circuit, $\quad I_g = \dfrac{V}{G + R} \Rightarrow R = \dfrac{V}{I_g} - G$

- **Current Sensitivity** The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_I = \frac{\alpha}{I} = \frac{NBA}{C}$$

- **Voltage Sensitivity** Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it.

$$S_V = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{S_I}{R} = \frac{NBA}{RC}$$

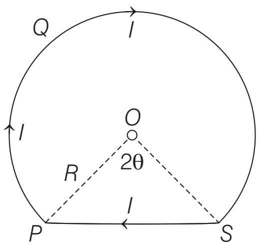
## DAY PRACTICE SESSION 1

# FOUNDATION QUESTIONS EXERCISE

**1** A loosely wound helix made of stiff wire is mounted vertically with the lower end just touching a dish of mercury. When a current from a battery is started in the coil through the mercury

(a) the wire oscillates
(b) the wire continues making contact
(c) the wire breaks contact just as current is passed
(d) the mercury will expand by heating due to passes of current

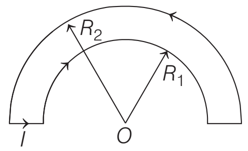
**2** A current $I$ flows through a closed loop as shown in figure.




The magnetic field induction at the centre $O$ is

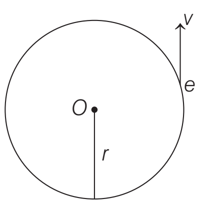
(a) $\frac{\mu_0 I}{4\pi R}\theta$ (b) $\frac{\mu_0 I}{4\pi R}(\theta + \sin\theta)$
(c) $\frac{\mu_0 I}{4\pi R}(\pi - \theta + \sin\theta)$ (d) $\frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta)$

**3** The magnetic induction at the centre $O$ in the figure as shown is



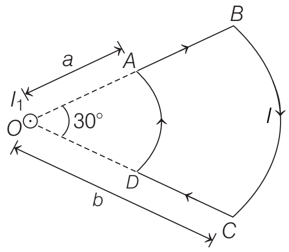

(a) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (b) $\frac{\mu_0 I}{4}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
(c) $\frac{\mu_0 I}{4}(R_1 - R_2)$ (d) $\frac{\mu_0 I}{4}(R_1 + R_2)$

**4** An electron moves in a circular orbit with a uniform speed $v$. It produces a magnetic field $B$ at the centre of the circle. The radius of the circle is proportional to




(a) $\frac{B}{v}$ (b) $\frac{v}{R}$ (c) $\sqrt{\frac{v}{B}}$ (d) $\sqrt{\frac{B}{v}}$

**5** The magnitude of the magnetic field ($B$) due to loop $ABCD$ at the origin ($O$) is → **AIEEE 2009**




(a) zero (b) $\frac{\mu_0 I(b-a)}{24ab}$
(c) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$ (d) $\frac{\mu_0 I}{4\pi}\left[2(b-a) + \frac{\pi}{3}(a+b)\right]$

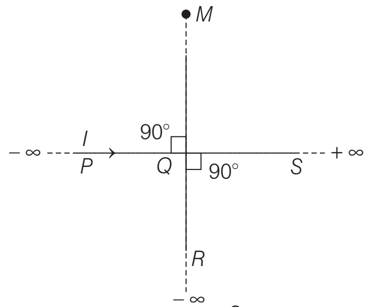
**6** A current $I$ flows in an infinity long wire with cross-section in the form of a semicircular ring of radius $R$. The magnitude of the magnetic induction along its axis is → **AIEEE 2012**

(a) $\frac{\mu_0 I}{2\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi R}$ (c) $\frac{\mu_0 I}{4\pi R}$ (d) $\frac{\mu_0 I}{\pi^2 R}$

**7** Two coaxial solenoids of different radii carry current $I$ in the same direction. Let $\mathbf{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\mathbf{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then, → **JEE Main 2015**

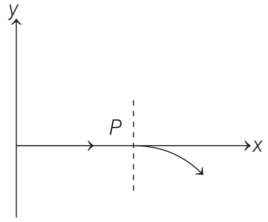
(a) $\mathbf{F}_1 = \mathbf{F}_2 = 0$
(b) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2$ is radially outwards
(c) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2 = 0$
(d) $\mathbf{F}_1$ is radially outwards and $\mathbf{F}_2 = 0$

**8** An infinitely long conductor $PQR$ is bent to form a right angle as shown. A current $I$ flows through $PQR$. The magnetic field due to this current at the point $M$ is $H_1$. Now another infinitely long straight conductor $QS$ is connected at $Q$, so that the current is $I/2$ in $OR$ as well as in $QS$, the current in $PQ$ remaining unchanged. The magnetic field at $M$ is now $H_2$. The ratio $H_1 : H_2$ is given by




(a) $\frac{1}{2}$ (b) 1 (c) $\frac{2}{3}$ (d) 2

**9** For a positively charged particle moving in a $xy$-plane initially along the $x$-axis,there is a sudden change in its path due to the presence of electric and/or magnetic fields beyond $P$. The curved path is shown in the $xy$-plane and is found to be non-circular. Which one of the following combinations is possible?




(a) $\mathbf{E} = 0, \mathbf{B} = b\hat{\mathbf{i}} + c\hat{\mathbf{k}}$ (b) $\mathbf{E} = a\hat{\mathbf{i}}; \mathbf{B} = c\hat{\mathbf{k}} + a\hat{\mathbf{i}}$
(c) $\mathbf{E} = 0; \mathbf{B} = c\hat{\mathbf{j}} + b\hat{\mathbf{k}}$ (d) $\mathbf{E} = a\hat{\mathbf{i}}; \mathbf{B} = c\hat{\mathbf{k}} + b\hat{\mathbf{j}}$

**10** A magnetic field $4 \times 10^{-3}$ kT exerts a force $(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times 10^{-10}$ N on a particle having a charge $10^{-9}$C and going on the $xy$-plane. The velocity of the particle is

(a) $-75\hat{\mathbf{i}} + 100\hat{\mathbf{j}}$ (b) $-100\hat{\mathbf{i}} + 75\hat{\mathbf{j}}$
(c) $25\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ (d) $2\hat{\mathbf{i}} + 25\hat{\mathbf{i}}$

**11** An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii $r_e, r_p, r_\alpha$ respectively, in a uniform magnetic field $B$. The relation between $r_e, r_p, r_\alpha$ is **→ JEE Main 2018**

(a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$ (c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$

**12** The cyclotron frequency of an electron gyrating in a magnetic field of 1 T is approximately

(a) 28 MHz (b) 280 MHz (c) 2.8 GHz (d) 28 GHz

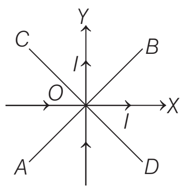
**13** A proton and an $\alpha$-particle enters a uniform magnetic field perpendicularly with the same speed. If proton takes 25 $\mu$s to make 5 revolutions, then the periodic time for the $\alpha$-particle would be

(a) 50 $\mu$s (b) 25 $\mu$s (c) 10 $\mu$s (d) 5 $\mu$s

**14** Two long conductors separated by a distance $d$ carry current $I_1$ and $I_2$ in the same direction. They exert a force $F$ on each other. Now the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is

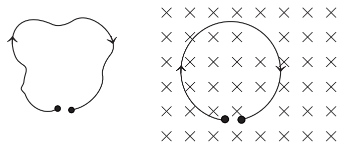
(a) $-2F$ (b) $\frac{F}{3}$ (c) $\frac{2F}{3}$ (d) $-\frac{F}{3}$

**15** Two very thin metallic wires placed along $X$ and $Y$-axes carry equal currents as shown in figure. $AB$ and $CD$ are lines at 45° with the axes with origin of axes at $O$. The magnetic field will be zero on the line




(a) $AB$
(b) $CD$
(c) segment $OB$ only of line $AB$
(d) segment $OC$ only of line $CD$

**16** A thin flexible wire of length $L$ is connected to two adjacent fixed points and carries a current $I$ in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength $B$ going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is **→ AIEEE 2011**

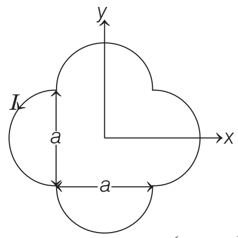


(a) $IBL$ (b) $\frac{IBL}{\pi}$ (c) $\frac{IBL}{2\pi}$ (d) $\frac{IBL}{4\pi}$

**17** Two parallel long wires $A$ and $B$ carry currents $I_1$ and $I_2 (< I_1)$. When $I_1$ and $I_2$ are in the same direction, the magnetic field at a point mid-way between the wires is 10 $\mu$T. If $I_2$ is reversed, the field becomes 30 $\mu$T. The ratio $I_1 / I_2$ is

(a) 1 (b) 3 (c) 2 (d) 4

**18** A loop carrying current $I$ lies in the $xy$- plane as shown in the figure. The unit vector $\hat{\mathbf{k}}$ is coming out of the plane of the paper. The magnetic moment of the current loop is **→ JEE Main (Online) 2013**

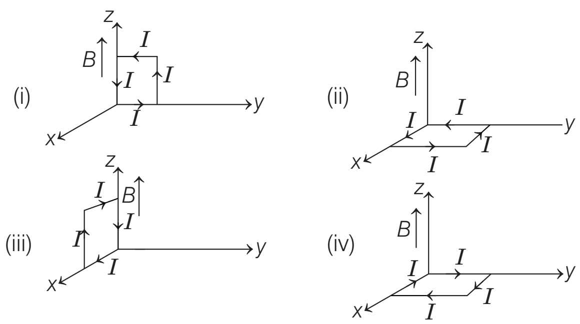



(a) $a^2 I\hat{\mathbf{k}}$ (b) $\left(\frac{\pi}{2} + 1\right) a^2 I\hat{\mathbf{k}}$
(c) $-\left(\frac{\pi}{2} + 1\right) a^2 I\hat{\mathbf{k}}$ (d) $(2\pi + 1) a^2 I\hat{\mathbf{k}}$

**19** Magnetic field at the centre of a circular loop of area $A$ is $B$. The magnetic moment of the loop will be

(a) $\frac{BA^2}{\mu_0 \pi}$ (b) $\frac{BA^{3/2}}{\mu_0 \pi}$ (c) $\frac{BA^{3/2}}{\mu_0 \pi^{1/2}}$ (d) $\frac{2BA^{3/2}}{\mu_0 \pi^{1/2}}$

**20** A rectangular loop of sides 10 cm and 5 cm carrying a current $I$ of 12 A is placed in different orientations as shown in the figures below. **→ JEE Main 2015**



If there is a uniform magnetic field of 0.3T in the positive $z$-direction, then in which orientations the loop would be in 1. stable equilibrium and 2. unstable equilibrium?

(a) (i) and (ii) respectively (b) (i) and (iii) respectively
(c) (ii) and (iv) respectively (d) (ii) and (iii) respectively

**21** When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is **→ JEE Main 2017 (Offline)**

(a) $2.045 \times 10^3$ Ω (b) $2.535 \times 10^3$ Ω
(c) $4.005 \times 10^3$ Ω (d) $1.985 \times 10^3$ Ω

**22** A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. The value of the resistance which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is **→ JEE Main 2016 (Offline)**

(a) 0.01 Ω (b) 2 Ω (c) 0.1 Ω (d) 3 Ω

**Direction** (Q. Nos. 23-28) *Each of these questions contains two statements : Statement* I *and Statement* II. *Each of these questions also has four alternative choice, only one of which is the correct answer. you have to select one of the codes* (*a*), (*b*), (*c*), (*d*) *given below*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation for Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation
(c) Statement I is true; Statement II is false
(d) Statement I is false; Statement II is true

**23** **Statement I** If a charged particle is projected in a region where **B** is perpendicular to velocity of projection, then the net force acting on the particle is independent of its mass.

**Statement II** The particle is performing uniform circular motion and net force acting on it is $\frac{mv^2}{r}$.

**24** **Statement I** A uniformly moving charged particle in a magnetic field, may follow a path along magnetic field lines.

**Statement II** The direction of magnetic force experienced by a charged particle is perpendicular to its velocity and **B**.

**25** **Statement I** The magnetic force experienced by a moving charged particle in a magnetic field is invariant in nature just like any other force.

**Statement II** Magnetic force experienced by a charged particle is given by $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$, where **v** is the velocity of charge particle w.r.t. frame of reference in which we are taking **F**.

**26** **Statement I** Cyclotron is a device which is used to accelerate the positive ion.

**Statement II** Cyclotron frequency depends upon the velocity.

**27** **Statement I** Magnetic field due to a infinite straight conductor varies inversely as the distance from it.

**Statement II** The lines of electric force due to a straight current carrying conductor are concentric circles.

**28** **Statement I** If a proton and α-particle enter a uniform magnetic field perpendicularly with the same speed, the time period of revolution of α-particle will be double than that of proton.

**Statement II** Time period of charged particle is given by $T = \frac{2\pi m}{Bq}$.

## DAY PRACTICE SESSION 2

# PROGRESSIVE QUESTIONS EXERCISE

**1** A cell is connected between two points of a uniformly thick circular conductor. $I_1$ and $I_2$ are the currents flowing in two parts of the circular conductor of radius $a$. The magnetic field at the centre of the loop will be

(a) zero (b) $\frac{\mu_0}{4\pi}(I_1 - I_2)$
(c) $\frac{\mu_0}{2a}(I_1 + I_2)$ (d) $\frac{\mu_0}{a}(I_1 + I_2)$

**2** A coil having $N$ turns is wound tightly in the form of a spiral with inner and outer radii $a$ and $b$ respectively. When a current $I$ passes through the coil, the magnetic field at the centre is

(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$
(c) $\frac{\mu_0 NI}{2(b-a)}\log_e \frac{b}{a}$ (d) $\frac{\mu_0 I^N}{2(b-a)}\log_e \frac{b}{a}$

**3** A particle of mass $m$ and charge $q$ moves with a constant velocity $v$ along the positive $x$-direction. It enters a region containing a uniform magnetic field $B$ directed along the negative $z$-direction, extending from $x = a$ to $x = b$. The minimum value of $v$ required, so that the particle can just enter the region $x > b$ is

(a) $qbB/m$ (b) $q(b-a)B/m$
(c) $qaB/m$ (d) $q(b+a)B/2m$

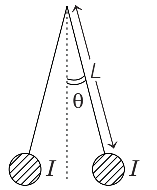
**4** The magnetic field normal to the plane of a wire of $n$ turns and radius $r$ which carries a current $I$ is measured on the axis of the coil at a small distance is from the centre of the coil. This is smaller than the magnetic field at the centre by the fraction

(a) $(2/3)r^2/h^2$ (b) $(3/2)r^2/h^2$
(c) $(2/3)h^2/r^2$ (d) $(3/2)h^2/r^2$

**5** A coil having $N$ turns is wound tightly in the form of a spiral with inner and outer radii $a$ and $b$ respectively. When a current $I$ passes through the coil, the magnetic field at the centre is **→ AIEEE 2012**

(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$
(c) $\frac{\mu_0 NI}{2(b-a)}\ln\frac{b}{a}$ (d) $\frac{\mu_0 I}{2(b-a)}\ln\frac{b}{a}$

**6** Two long current carrying thin wires, both with current $I$, are held by insulating threads of length $L$ and are in equilibrium as shown in the figure, with threads making an angle $\theta$ with the vertical. If wires have mass $\lambda$ per unit length, then the value of $I$ is
($g$ = gravitational acceleration)




**→ JEE Main 2015**

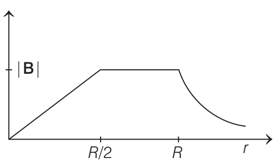
(a) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (b) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$
(c) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$ (d) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$

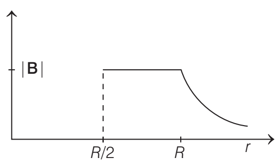
**7** Two identical wires $A$ and $B$, each of length $l$, carry the same current $I$. Wire $A$ is bent into a circle of radius $R$ and wire $B$ is bent to form a square of side $a$. If $B_A$ and $B_B$ are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

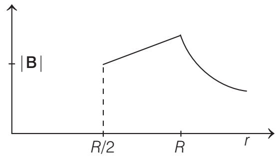
**→ JEE Main 2016 (Offline)**

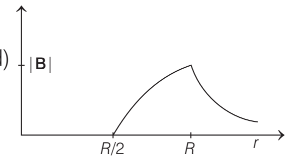
(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

**8** An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius $R$ carries a uniform current density along its length. The magnitude of the magnetic field, $|\mathbf{B}|$ as a function of the radial distance $r$ from the axis is best represented by **→ JEE Main (Online) 2013**

(a)


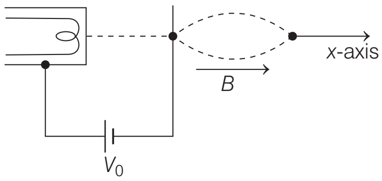

(b)



(c)



(d)


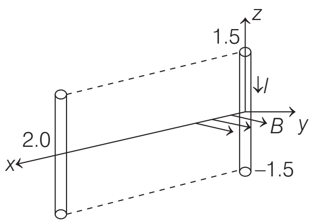

**9** Electrons emitted with negligible speed from an electron gun are accelerated through a potential difference $V_0$ along the $x$-axis. These electrons emerge from a narrow hole into a uniform magnetic field of strength $B$ directed along $x$-axis.

Some electrons emerging at slightly divergent angles as shown. These paraxial electrons are refocussed on the $x$-axis at a distance.



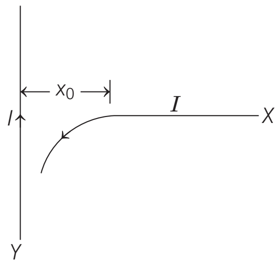

(a) $\frac{8\pi^2 mV_0}{3eB}$ (b) $\sqrt{\frac{8\pi^2 mV_0}{eB^2}}$ (c) $\sqrt{\frac{2\pi^2 mV_0}{eB^2}}$ (d) $\sqrt{\frac{4\pi^2 mV_0}{eB^2}}$

**10** A conductor lies along the $z$-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure). For a field $\mathbf{B} = 3.0\times10^{-4}e^{-0.2x}a_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = 0$ in $5\times10^{-3}$ s. Assume parallel motion along the $x$-axis. **→ JEE Main 2014**




(a) 1.57 W (b) 2.97 W (c) 14.85 W (d) 29.7 W

**11** A long straight wire carries a current $I$. A particle of charge $+q$ and mass $m$ is projected with a speed $v$ from a distance $x_0$ as shown. The minimum separation between the wire and particle is

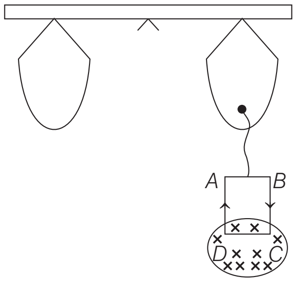



(a) $x_0 e^{-\frac{2\pi mv}{\mu_0 qI}}$ (b) $x_0 e^{-\frac{mv2\pi x_0}{\mu_0 qI}}$ (c) $x_0 e^{-\frac{2\pi mv}{qI}}$ (d) zero

**12** A current carrying circular loop of radius $R$ is placed in the $xy$-plane with centre at the origin. Half of the loop with $x > 0$ is now bent, so that it now lies in the $yz$-plane.

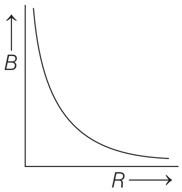
(a) The magnitude of magnetic moment now diminishes
(b) The magnetic moment does not change
(c) The magnitude of B at $(0, 0, z)$, $z >> R$ increases
(d) The magnitude of B at$(0, 0, z)$, $z >> R$ is unchanged

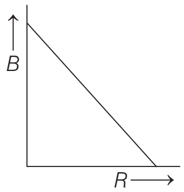
**13** A 100 turn rectangular coil $ABCD$ (in $xy$-plane) is hung from one arm of a balance (shown in figure). A mass 500 g is added to the other arm to balance the weight of the coil. A current of 4.9 A passes through the coil and a constant magnetic field of 0.2 T acting inward (in $xz$-plane) is switched on such that only arm $CD$ of length 1cm lies in the field. How much additional mass $m$ must be added to regain the balance?

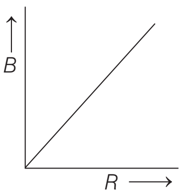


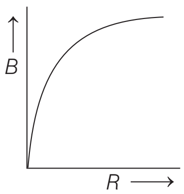

(a) 2 g (b) 1 g (c) 0.75 g (d) 1.5 g

**14** A charge $Q$ is uniformly distributed over the surface of non-conducting disc of radius $R$. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity $\omega$. As a result of this rotation, a magnetic field of induction $B$ is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure. **→ JEE Main (Online) 2013**

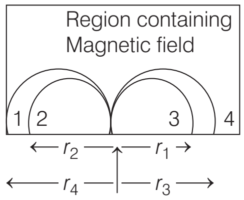
(a)



(b)



(c)



(d)



**15** A beam consisting of four types of ions $a,b,c$ and $d$ enters a region that contains a uniform magnetic field as shown in figure. The field is perpendicular to the plane of the paper, but its precise direction is not given. All ions in the beam travel with the same speed.




The table below gives the masses and charges of the ions

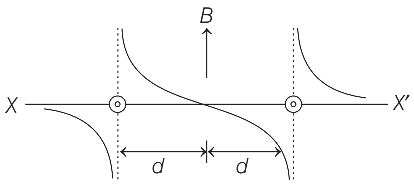
| ION | MASS | CHARGE |
|---|---|---|
| A | $2m$ | $+e$ |
| B | $4m$ | $-e$ |
| C | $2m$ | $-e$ |
| D | $m$ | $+e$ |

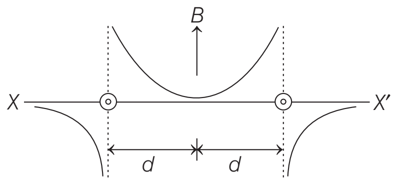
The ions fall at different positions 1,2,3 and 4 as shown. Correctly match the ions with respective falling positions

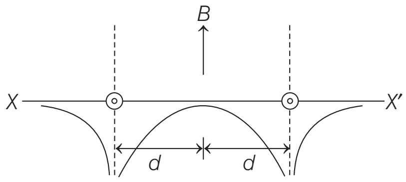
| Column I | Column II |
|---|---|
| A | 1 |
| B | 2 |
| C | 3 |
| D | 4 |

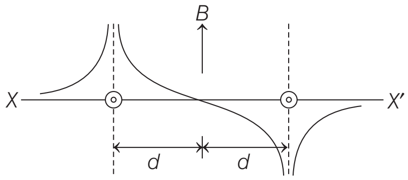
| | A | B | C | D |
|---|---|---|---|---|
| (a) | 4 | 3 | 2 | 1 |
| (b) | 1 | 2 | 3 | 4 |
| (c) | 4 | 1 | 2 | 3 |
| (d) | 3 | 4 | 1 | 2 |

**16** Two long parallel wires are at a distance $2d$ apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line $XX'$ is given by **→ AIEEE 2010**

(a)



(b)



(c)



(d)


# ANSWERS

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| SESSION 1 | **1** (a) | **2** (d) | **3** (a) | **4** (c) | **5** (b) | **6** (d) | **7** (a) | **8** (c) | **9** (b) | **10** (a) |
| | **11** (b) | **12** (d) | **13** (c) | **14** (c) | **15** (a) | **16** (c) | **17** (c) | **18** (b) | **19** (d) | **20** (c) |
| | **21** (d) | **22** (a) | **23** (c) | **24** (d) | **25** (d) | **26** (c) | **27** (b) | **28** (a) | | |
| SESSION 2 | **1** (a) | **2** (c) | **3** (b) | **4** (d) | **5** (c) | **6** (b) | **7** (d) | **8** (d) | **9** (b) | **10** (b) |
| | **11** (a) | **12** (a) | **13** (b) | **14** (a) | **15** (c) | **16** (a) | | | | |

# Hints and Explanations

## SESSION 1

**1** Loosely wound helix get compressed when possess current. So, connection of mercury lost and demagnetisation takes place. So, result a oscillatory motion.

**2** $B_0 = B_{PQS} + B_{SP}$

$$= \frac{\mu_0}{4\pi}\frac{I}{R}(2\pi - 2\theta) + \frac{\mu_0}{4\pi}\frac{1}{R\cos\theta}(\sin\theta + \sin\theta)$$

$$= \frac{\mu_0 I}{2\pi R}(\pi - \theta + \tan\theta)$$

**3** Magnetic field due to straight parts of wire at point $O = 0$. Field due to a semicircular current loop of radius $R_1$, $B_1 = \frac{\mu_0 I}{2R_1}\left(\frac{\pi}{2\pi}\right) = \frac{\mu_0 I}{4R_1}$ into the plane of paper.

Field due to semicircular current loop of radius $R_2$, $B_2 = \frac{\mu_0 I}{4R_2}$ outside the plane of paper.

$\therefore$ Net field $B = B_1 - B_2$

$$= \frac{\mu_0 I}{4}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

**4** Equivalent current, $I = \frac{ev}{2\pi r}$

Hence, magnetic field at centre of circle

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0}{2r}\cdot\frac{ev}{2\pi r} = \frac{\mu_0 ev}{4\pi r^2}$$

$$\Rightarrow \quad r = \sqrt{\frac{\mu_0 ev}{4\pi B}} \Rightarrow r \propto \sqrt{\frac{v}{B}}$$

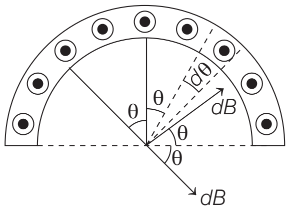
**5** Net magnetic field due to loop $ABCD$ at $O$ is

$$B = B_{AB} + B_{BC} + B_{CD} + B_{OA}$$

$$= 0 + \frac{\mu_0 I}{4\pi a}\times\frac{\pi}{6} + 0 - \frac{\mu_0 I}{4\pi b}\times\frac{\pi}{6}$$

$$= \frac{\mu_0 I}{24a} - \frac{\mu_0 I}{24b} = \frac{\mu_0 I}{24ab}(b - a)$$

**6** Consider the wire to be made up of large number of thin wires of infinite length. Consider such wire of thickness $dl$ subtending an angle $d\theta$ at centre.




Current through this wire, $dI = \frac{d\theta}{\pi}I$

$\therefore$ Magnetic field at centre due to this portion,

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{2dI}{R} = \frac{\mu_0 I}{2\pi^2 R}d\theta$$

Net magnetic field at the centre

$$B = \int_{-\pi/2}^{\pi/2} dB\cos\theta$$

$$= \frac{\mu_0 I}{2\pi^2 R}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{\pi^2 R}$$

**7** Consider the two coaxial solenoids. Due to one of the solenoids magnetic field at the centre of the other can be assumed to be constant. Due to symmetry, forces on upper and lower part of a solenoid will be equal and opposite and hence resultant is zero.

Therefore, $\mathbf{F}_1 = \mathbf{F}_2 = 0$

**8** Magnetic field at any point lying on the current carrying straight conductor is zero.

Here, $H_1$ = Magnetic field at $M$ due to current in $PQ$

$H_2$ = Magnetic field at $M$ due to $QR$ + magnetic field at $M$ due to $QS$ + magnetic field at $M$ due to $PQ$

$$= 0 + \frac{H_1}{2} + H_1 = \frac{3}{2}H_1$$

$$\Rightarrow \quad \frac{H_1}{H_2} = \frac{2}{3}$$

**9** Electric field can deviate the path of the particle in the shown direction only when it is along negative $y$-direction. In the given option **E** is either zero or along $x$-direction. Hence, it is the magnetic field which is really responsible for its curved path. Options (a) and (c) cannot be accepted as the path will be helix in that case (when the velocity vector makes an angle other than 0°,180 or 90° with the magnetic field, path is a helix). Option (d) is wrong because in that case component of net force on the particle also comes in $k$ direction which is not acceptable as the particle is moving in $xy$-plane. Only in option (b) the particle can move in $xy$-plane.

In option (d) : $\mathbf{F}_{net} = q\mathbf{E} + q(\mathbf{v}\times\mathbf{B})$

Initial velocity is along x-direction. So let $\mathbf{v} = v\hat{\mathbf{i}}$

$$\therefore \quad \mathbf{F}_{net} = qa\hat{\mathbf{i}} + q[(v\hat{\mathbf{i}})\times(c\hat{\mathbf{k}} + b\hat{\mathbf{j}})]$$

$$= qa\hat{\mathbf{i}} - qvc\hat{\mathbf{j}} + qvb\hat{\mathbf{k}}$$

In option (b),

$$\mathbf{F}_{net} = q(a\hat{\mathbf{i}}) + q[(v\hat{\mathbf{i}})\times(c\hat{\mathbf{k}} + a\hat{\mathbf{i}})]$$

$$= qa\hat{\mathbf{i}} + qv\hat{\mathbf{j}}$$

**10** From Lorentz force, $F = q(v\times B)$

Given, $F = (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\times 10^{-10}$ N,

$q = 10^{-9}$C, $B = 4\times 10^{-3}\,\hat{\mathbf{k}}$T

$$\therefore \quad (4\hat{\mathbf{i}} + 3\hat{\mathbf{j}})\times 10^{-10} = 10^{-9}(a\hat{\mathbf{i}} + b\hat{\mathbf{j}})\times(4\times 10^{-3})$$

Solving, we get

$a = -75, b = 100 \Rightarrow v = -75\hat{\mathbf{i}} + 100\hat{\mathbf{j}}$

**11** From $Bqv = \frac{mv^2}{r}$, we have

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

where, $K$ is the kinetic energy.
As, kinetic energies of particles are same;

$$r \propto \frac{\sqrt{m}}{q}$$

$$\Rightarrow \quad r_e : r_p : r_\alpha = \frac{\sqrt{m_e}}{e} : \frac{\sqrt{m_p}}{e} : \frac{\sqrt{4m_p}}{2e}$$

Clearly, $r_p = r_\alpha$ and $r_e$ is least $[\because m_e < m_p]$

So, $\quad r_e < r_p = r_\alpha$

**12** Cyclotron frequency,

$$\nu = \frac{Bq}{2\pi m} = \frac{1 \times 1.6 \times 10^{-19}}{2\pi \times 9.1 \times 10^{-31}}$$

$$= 2.8 \times 10^{10}\ \text{Hz} = 28\ \text{GHz}$$

**13** Time taken by to make one revolution

$$= \frac{25}{5} = 5\,\mu\text{s}$$

As $T = \frac{2\pi m}{qB}$; so $\frac{T_2}{T_1} = \frac{m_2}{m_1} \times \frac{q_1}{q_2}$

or $T_2 = T_1 \frac{m_2 q_1}{m_1 q_2} = \frac{5 \times 4m_1}{m_1} = \frac{q}{2q}$

$$= 10\,\mu\text{s}$$

**14** Initial force $F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{d}$ (attractive)

and final force $F' = \frac{\mu_0}{4\pi} \cdot \frac{2(2I_1)I_2}{(3d)}$

(repulsive) $\Rightarrow F' = \frac{2}{3} F$

**15** Along the line $AB$, magnetic field due to two wires is equal but in mutually opposite directions (as per right hand thumb rule). Hence, net magnetic field will be zero along the line $AB$.

**16**


$$2T \sin\left(\frac{d\theta}{2}\right) = IdlB$$

$$\Rightarrow \quad 2T \frac{d\theta}{2} = IRd\theta B \quad [\because dl = Rd\theta]$$

$$\Rightarrow \quad T = BIR = \frac{BIL}{2\pi} \quad [\because L = 2\pi R]$$

**17** $\frac{\mu_0}{4\pi} \frac{2I_1}{r} - \frac{\mu_0}{4\pi} \frac{2I_2}{r} = 10\,\mu\text{T}$

$$\frac{\mu_0 2I_1}{4\pi r} + \frac{\mu_0}{4\pi} \frac{2I_2}{r} = 30\,\mu\text{T}$$

On solving, $\quad I_1 = 20\ \text{A}$ and $I_2 = 10\ \text{A}$

So, $\quad I_1 / I_2 = 2$

**18** As, $M = I \times$ Area of loop

$$= I \times \left[a^2 + \frac{\pi a^2}{4 \times 2} \times 4\right] \mathbf{k}$$

$$= I \times a^2 \left[\frac{\pi}{2} + 1\right] \mathbf{k} = \left(\frac{\pi}{2} + 1\right) a^2 I\, \mathbf{k}$$

**19** As, $B = \frac{\mu_0}{4\pi} \frac{2\pi I}{r} = \frac{\mu_0 I}{2r}$

$$\Rightarrow \quad I = \frac{2Br}{\mu_0}$$

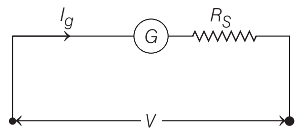
Also $A = \pi r^2$ or $r = \left(\frac{A}{\pi}\right)^{1/2}$

Magnetic moment, $M = IA = \frac{2Br}{\mu_0} A$

$$= \frac{2BA}{\mu_0} \times \left(\frac{A}{\pi}\right)^{1/2} = \frac{2BA^{3/2}}{\mu_0 \pi^{1/2}}$$

**20** Since, **B** is uniform only torque acts on a current carrying loop.
As, $\tau = \mathbf{M} \times \mathbf{B} \Rightarrow |\tau| = |\mathbf{M}||\mathbf{B}| \sin\theta$
For orientation shown in figure (ii),
$\theta = 0^0, \tau = 0$ (stable equilibrium)
and for figure (iv), $\theta = \pi, \tau = 0$
(unstable equilibrium)

**21** Suppose a resistance $R_s$ is connected in series with galvanometer to convert it into voltmeter.

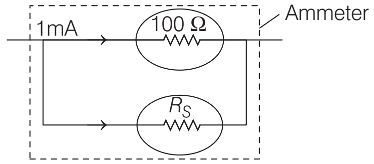



$$I_g (G + R_s) = V \Rightarrow R = \frac{V}{I_g} - G$$

$\Rightarrow \quad R = 1985 = 1.985\ \text{k}\Omega$

or $\quad R = 1.985 \times 10^3\ \Omega$

**22** Maximum voltage that can be applied across the galvanometer coil
$= 100\,\Omega \times 10^{-3}\ \text{A} = 0.1\ \text{V}$




If $R_s$ is the shunt resistance, then

$$R_s \times 10\,\text{A} = 0.1\ \text{V}$$

$\Rightarrow \quad R_s = 0.01\,\Omega$

**23** In this case, the charged particle performs uniform circular motion and magnetic force is providing the necessary centripetal force,

i.e. $\quad \frac{mv^2}{r} = qvB$

$\frac{mv^2}{r}$ is not the force acting on charged particle it is simply equal to net force acting on the particle.

**24** Statement I is false, as **B** is perpendicular to **F**, so particle cannot follow magnetic field lines (tangent to which gives the direction of magnetic field).

**25** Statement I is false and Statement II is true. Magnetic force may have different value in different frames of reference, that's why it is not invariant in nature.

**26** Cyclotron frequency is given by

$$\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$$

It is obvious that cyclotron frequency does not depend upon velocity of charged particle.

**27** The magnetic field at a point due to current flowing through an infinitely long conductor is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2I}{a}.$$

where, $a$ is the distance of that point from conductor. Now, according to right hand thumb rule it follows that magnetic field is in the form of concentric circles, whose centres lie on the straight conductor.

**28** We know that,

$$T \propto \frac{m}{q} \qquad \left[\because T = \frac{2\pi m}{Bq}\right]$$

For $\alpha$-particle,

$$T_\alpha \propto \frac{4m}{2q}$$

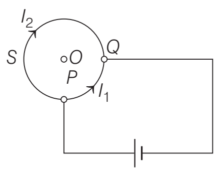
For proton,

$$T_{p^+} \propto \frac{m}{q}$$

So, $\quad 2T_p = T_\alpha$

## SESSION 2

**1** The resistance of the portion $PRQ$ will be
$R_1 = l_1 \rho$



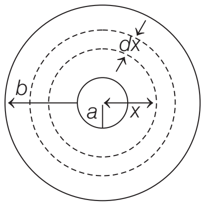

Resistance of the portion $PSQ$ will be
$R_2 = l_2 \rho$
Potential difference across $P$ and $Q$
$= I_1 R_1 = I_2 R_2$

$$I_1 l_1 \rho = I_2 l_2 \rho \text{ or } I_1 l_1 = I_2 l_2 \quad \text{...(i)}$$

Magnetic field induction at the centre $O$ due to currents through circular conductors $PRQ$ and $PSQ$ will be
$= B_1 - B_2$

$$= \frac{\mu_0}{4\pi} \frac{I_1 l_1 \sin 90^\circ}{r^2} - \frac{\mu_0}{4\pi} \frac{I_2 l_2 \sin 90^\circ}{r^2} = 0$$

**2** Refer to the figure, number of turns in $dx$, $n = \frac{N \cdot dx}{b - a}$.



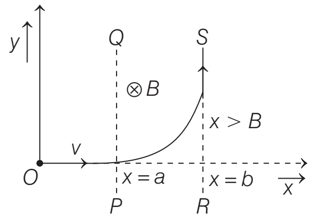

The magnetic field induction at the centre $O$ due to current $I$ through the entire spiral is

$$B = \int_a^b \frac{\mu_0}{4\pi} \frac{2\pi n I dx}{x}$$

$$= \frac{\mu_0}{4\pi} 2\pi \int_a^b \left(\frac{N}{b-a}\right) \frac{dx}{x} I$$

$$= \frac{\mu_0}{4\pi} \frac{NI}{(b-a)} (\log_e x)_a^b = \frac{\mu_0}{2} \frac{NI}{(b-a)} \log_e \frac{b}{a}$$

**3** In the figure, the $z$-axis points out of the paper and the magnetic field is directed into the paper, existing in the region between $PQ$ and $RS$. The particle moves in a circular path of radius $r$ in the magnetic field. It can just enter the region $x > b$ for $r \geq (b - a)$.




Now, $r = \frac{mv}{qB} \geq (b - a)$

or $v \geq \frac{q(b-a)B}{m} \Rightarrow v_{\min} = \frac{q(b-a)B}{m}$

**4** $B_1 = \frac{\mu_0}{4\pi} \frac{2\pi n I}{r}$ and $B_2 = \frac{\mu_0}{4\pi} \frac{2\pi n I r^2}{(r^2 + h^2)^{3/2}}$

So, $\frac{B_2}{B_1} = \left(1 + \frac{h^2}{r^2}\right)^{-\frac{3}{2}}$

Fractional decrease in the magnetic field will be $= \frac{B_1 - B_2}{B_1} = \left(1 - \frac{B_2}{B_1}\right)$

$$= \left[1 - \left(1 + \frac{h^2}{r^2}\right)^{-3/2}\right]$$

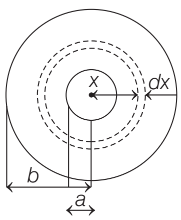
$$= 1 - \left(1 - \frac{3}{2} \cdot \frac{h^2}{r^2}\right) = \frac{3}{2} \frac{h^2}{r^2}$$

**5** Number of turns per unit width $= \frac{N}{b - a}$

Consider an elemental ring of radius $x$ and thickness $dx$.

Number of turns in the ring

$$= dN = \frac{Ndx}{b-a}$$




Magnetic field at the centre due to the ring element

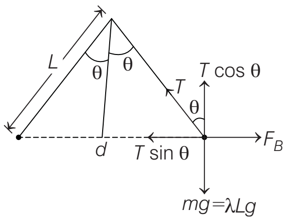
$$dB = \frac{\mu_0 (dN) I}{2x} = \frac{\mu_0 I}{2} \cdot \frac{Ndx}{(b-a)} \cdot \frac{1}{x}$$

∴ Field at the centre

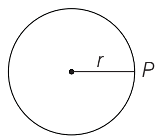
$$= \int dB = \frac{\mu_0 NI}{2(b-a)} \int_a^b \frac{dx}{x}$$

$$= \frac{\mu_0 NI}{2(b-a)} \ln \frac{b}{a}$$

**6** Consider free body diagram of the wire. As the wires are in equilibrium, they must carry current in opposite direction.




Here, $F_B = \frac{\mu_0 I^2 L}{2\pi d}$, where, $L$ is length of each wire are $d$ is separation between wires.




From figure, $d = 2L \sin\theta$

$T \cos\theta = mg = \lambda L g$

(in vertical direction) ...(i)

$$T \sin\theta = F_B = \frac{\mu_0 I^2 L}{4\pi L \sin\theta}$$

(In horizontal direction) ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{T \sin\theta}{T \cos\theta} = \frac{\mu_0 I^2 L}{4\pi L \sin\theta \times \lambda L g}$$

$$\therefore I = \sqrt{\frac{4\pi\lambda L g \sin^2\theta}{\mu_0 \cos\theta}} = 2\sin\theta \sqrt{\frac{\pi\lambda L g}{\mu_0 \cos\theta}}$$

**7** Magnetic field in case of circle of radius $R$, we have

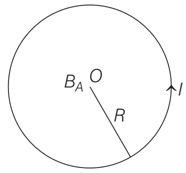
$$B_A = \frac{\mu_0 I}{2R}$$

As, $2\pi R = l$ ($l$ is length of a wire)

$$R = \frac{l}{2\pi}$$

$$\Rightarrow B_A = \frac{\mu_0 I}{2 \times \frac{l}{2\pi}}$$

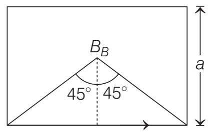
$$= \frac{\mu_0 I \pi}{l} \quad \text{...(i)}$$




Magnetic field in case of square of side $a$, we get

$$B_B = 4 \times \frac{\mu_0}{4\pi} \times \frac{I}{\left(\frac{a}{2}\right)} \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow B_B = \frac{4I\mu_0}{\pi a\sqrt{2}} = \frac{\mu_0 2\sqrt{2} I}{a\pi}$$



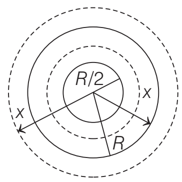

As, $4a = l$, $a = \frac{l}{4} \Rightarrow B_B = \frac{8\sqrt{2}\,\mu_0 I}{\pi l}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

**8** ***Case* I** $x < \frac{R}{2}$,

$|\mathbf{B}| = 0$




***Case* II** $\frac{R}{2} \leq x < R$

$$\int \mathbf{B} \cdot d\,\mathbf{l} = \mu_0 I$$

$$|\mathbf{B}| 2\pi x = \mu_0$$

$$|\mathbf{B}| = \frac{\mu_0 J}{2x}\left(x^2 - \frac{R^2}{4}\right) \left[\pi x^2 - \pi\left(\frac{R}{2}\right)^2\right] J$$

***Case* III** $x \geq R$

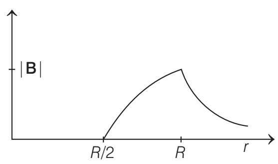
$$\int \mathbf{B} \cdot d\,\mathbf{l} = \mu_0 I$$

$$|\mathbf{B}| 2\pi x = \mu_0 \left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right] J$$

$$|\mathbf{B}| = \frac{\mu_0 J}{2x} \frac{3}{2} R^2,$$

$$|\mathbf{B}| = \frac{3\mu_0 J R^2}{8x}$$

So,



**9** The electrons will be refocussed after distance equal to pitch.

$$\text{Pitch} = V_x T = V \frac{2\pi m}{eB}$$

$$= \sqrt{\frac{2eV_0}{m}} \cdot \frac{2\pi m}{eB}$$

$$= \sqrt{\frac{8\pi^2 m V_0}{eB^2}} \quad \left[\because eV_0 = \frac{1}{2}mv^2\right]$$

**10** When force exerted on a current carrying conductor

$$F_{ext} = BIL$$

$$\text{Average power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$P = \frac{1}{t}\int_0^2 F_{ext.} \cdot dx = \frac{1}{t}\int_0^2 B(x)IL\, dx$$

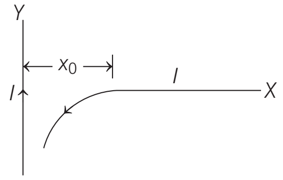
$$= \frac{1}{5 \times 10^{-3}}\int_0^2 3 \times 10^{-4} e^{-0.2x} \times 10 \times 3\, dx$$

$$= 9\,[1 - e^{-0.4}]$$

$$= 9\left[1 - \frac{1}{e^{0.4}}\right] = 2.967 \approx 2.97 \text{ W}$$

**11** $F = q(\hat{\mathbf{i}}\, v_x + \hat{\mathbf{j}}\, v_y) \times \left[\frac{\mu_0 I}{2\pi x}\hat{\mathbf{k}}\right]$

$$= \hat{\mathbf{j}} q\, v_x \frac{\mu_0 I}{2\pi x} - \hat{\mathbf{i}} q\, v_y \frac{\mu_0 I}{2\pi x}$$




$$\therefore \quad a_x = \frac{F_x}{m} = -\frac{\mu_0 I q v_y}{2\pi x m}$$

Also, $a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx} \cdot \frac{dx}{dt} = \frac{v_x\, dv_x}{dx}$

Since, $v_x^2 + v_y^2 = v^2$

or $\quad 2v_x\, dv_x + 2v_y dv_y = 0$

$\Rightarrow \quad v_x\, dv_x = -v_y dv_y$

Hence,

$$\frac{v_x\, dv_x}{dx} = -\frac{v_y dv_y}{dx} = -\frac{\mu_0 I q\, v_y}{2\pi\, x m}$$

$$\Rightarrow \quad \frac{dx}{x} = \frac{dv_y 2\pi\, m}{\mu_0 q I}$$

Initially, $x = x_0$ and $v_y = 0$

At minimum separation $v_x = 0$, $v_y = v$

Thus, $\int_{x_0}^{x} \frac{dx}{x} = \frac{2\pi m}{\mu_0 q I}\int_0^{-v} dv_y$

$$\Rightarrow \quad \log\frac{x}{x_0} = -\frac{2\pi\, m\, v}{\mu_0 q I}$$

$$\Rightarrow \quad x = x_0 e^{\frac{-2\pi\, m\, v}{\mu_0\, q I}}$$

**12** For a circular loop of radius $R$, carrying current $I$ in $xy$-plane, the magnetic moment $M = I \times \pi R^2$. It acts perpendicular to the loop along $z$-direction. When half of the current loop is bent in $yz$- plane, then magnetic moment due to half current loop in $xy$-plane, $M_1 = I\,(\pi R^2/2)$ acting along $z$-direction. Magnetic moment due to half current loop in $yz$-plane, $M_2 = I\,(\pi R^2/2)$ along $x$-direction.

Effective magnetic moment due to entire bent current loop,

$$M' = \sqrt{M_1^2 + M_2^2}$$

$$= \sqrt{(I\pi R^2/2)^2 + (I\pi R^2/2)^2}$$

$$= \frac{I\pi R^2}{2}\sqrt{2} < M$$

i.e. magnetic moment diminishes

The magnitude of **B** at a point on the axis of loop, distance $z$ from the centre of current loop in $xy$-plane is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi / R^2}{(R^2 + z^2)^{3/2}}$$

The magnitude of **B** at a point distance $z$ from the centre of bent current loop, whose half part is in $xy$-plane and half part is in $yx$- plane, is

$$B = \sqrt{\left[\frac{\mu_0}{4\pi}\frac{\pi / R^2}{(R^2 + z^2)^{3/2}}\right]^2 + \left[\frac{\mu_0}{4\pi}\frac{\pi / R^2}{(R^2 + z^2)^{3/2}}\right]^2}$$

$$= \frac{\mu_0}{4\pi}\frac{\pi / R^2}{(R^2 + z^2)^{3/2}}\sqrt{2} < B$$

**13** Let the mass of coil $= M$

Mass added other arm $= 500$ g $= 500 \times 10^{-3}$ kg

Mass of coil = Mass in other arm (for balancing)

$$M = 500 \times 10^{-3} \text{ kg}$$

$$M = 0.5 \text{ kg}$$

When the current is switched on,

Current, $I = 4.9$ A

Magnetic field $B = 0.2$ T ($xz$-plane)

Length of arm $CD = 1$ cm,

Mass added to balance $= m$

Let $F$ be the force due to magnetic field.

The direction of magnetic field is inward in $xz$-plane, the length vector is left, so by using the Fleming's left hand rule, the direction of force is downwards in the plane of paper

$$F = I\,(l \times B) = 4.9\,(0.01 \times 0.2 \sin 90°)$$

$$F = 4.9 \times 0.01 \times 0.2 \quad \text{...(i)}$$

For balancing,

mass of coil $\times\, g$ + force due to magnetic field

$$= 500 \times 10^{-3} g + m \times g$$

$$0.5 \times 9.8 + 4.9 \times 0.01 \times 0.2 = 500 \times 10^{-3} \times 9.8 + m \times 9.8$$

$$9.8\,(0.5 + 0.001) = 9.8\,(0.5 + m)$$

[From Eq. (i)]

$$m = 0.001 \text{ kg} = 1 \text{ g}$$

Thus, 1 g of mass must be added to regain the balance.

**14** $dB = \frac{\mu_0 (dq)}{2r}\left(\frac{\omega}{2\pi}\right)$

$$B = \int dB = \frac{\mu_0 \omega}{4\pi} \cdot \frac{Q}{\pi R^2} 2\pi \int_0^R \frac{r dr}{r}$$

$$B = \frac{\mu_0 \omega Q}{2\pi R^2} \cdot R, \quad B = \frac{\mu_0 \omega Q}{2\pi R} \Rightarrow B \propto \frac{1}{R}$$

**15** A $\rightarrow$ 4, B $\rightarrow$ 1, C $\rightarrow$ 2, D $\rightarrow$ 3

$$R = mv/qB$$

$$R_B > R_A$$

and $R_A = R_C$ (in opposite sense)

and $R_D$ is smallest.

**16** The magnetic field in between because of each will be in opposite direction

$$B_{\text{in between}} = \frac{\mu_0 I}{2\pi x}\hat{\mathbf{j}} - \frac{\mu_0 I}{2\pi\,(2d - x)}(-\hat{\mathbf{j}})$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} - \frac{1}{2d - x}\right](\hat{\mathbf{j}})$$

at $x = d$, $B_{\text{in between}} = 0$

For $x < d$, $B_{\text{in between}} = (\hat{\mathbf{j}})$ and

For $x > d$, $B_{\text{in between}} = (-\hat{\mathbf{j}})$

Towards $x$, net magnetic field will add up and direction will be $(-\mathbf{j})$ and towards $x'$, net magnetic field will add up and direction will be $(\mathbf{j})$.